

Tokai Research Institute for Environment and Sustainability

TRIES Discussion Paper Series

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April 4, 2022

DP2022-01



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Abstract

Extended producer responsibility (EPR) requires producers to be responsible either financially or physically for proper waste treatment. It is often argued that the price signal of waste induced by financial EPR gives producers proper motivation for adopting design for environment (DfE) or eco-design, leading an economy to a preferable situation, compared to the case in which such responsibility is not required. This paper examines this argument in a rigorous way by means of a dynamic system, and demonstrates that the price signal of financial EPR fails in playing such a role in a certain circumstance.

Keywords: End-of-life product (ELP), secondary material, extended producer responsibility (EPR), design for environment (DfE), eco-design

JEL Classification: E11, O41, Q53

*I am grateful to Professor Ken-Ichi Akao for his comments on the earlier draft of this paper.

1 Introduction

Circulative use of resources is a new trend in advanced countries, and extended producer responsibility (EPR) is considered to be one of the most important tools for fulfilling that purpose (OECD 2016). As is articulated in OECD (2001), EPR is designed to promote design for environment (DfE) or eco-design¹, accelerating circulative use of resources. It is often insisted that full-costs internalization of waste treatment required by extended producer responsibility (EPR) gives proper motivation for promoting DfE or eco-design in the right direction; if producers are obliged to owe full costs of waste treatment of their own products, they are given incentives to change the design of products so that reuse and recycling become easier and the waste treatment costs are reduced (OECD 2016). To the best of the author's knowledge, however, this argument has not been discussed rigorously by economic researchers, and I would like to address the problem and solve it by means of a Sraffian type of economic model.

Actually, there are not so many comparable studies to the present paper. Certainly, the relationship between EPR and DfE/eco-design is analytically explored by researchers in main-stream economics like Calcott and Walls (2000), Eichner and Pethig (2001), Fullerton and Wu (1998), Walls and Palmer (2001), Subramanian et al (2009) and so on. Yet, they do not analyse DfE/eco-design or green design in a dynamic phase.

Insofar as I know, dynamic analysis in this field is quite limited. Weis et al. (2010) deals with energy efficiency of large appliances by means of the experience curve approach in a dynamic model and analyses a policy effect on energy consumption of large appliances by means of an empirical approach.

Brouillat et al (2012) takes seemingly a similar approach to the present study in the sense that it explores the dynamic effect of EPR on eco-design. By means of a simulation model, they analyse how eco-design is affected by various actors' decisions in a product chain. Their emphasis is more on innovation rather than choice of technique on DfE/eco-design.

DfE/eco-design is often referred to in the case of containers and packages. Dace et al (2014) takes up this theme and studies policy effects on material efficiency of packages by means of system dynamics. An interesting result is that packaging tax, being coupled with other policy options, is very effective in enhancing material efficiency. The result of effectiveness of taxation is shared by Brouillat et al (2012).

Chen, Y.J. and J-B. Sheu (2009) explores how the recyclability of product green design is promoted, using a sophisticated differential game model. They demonstrate that, by adopting financial incentive and increasing stringent regulation, the government can affect producers' incentive so that product recyclability is enhanced.

The present paper shares a nature of dynamic analysis of a policy effect on DfE/eco-design with the above studies. However, the present dynamic model is quite different from the above studies in several senses: first of all, it is written in the Sraffian spirit (Sraffa 1960). Thus, an inter-industrial relationship is emphasized and its dynamic nature is explored. Secondly, whether and how a short-run competitive equilibrium converges to a long-run competitive one is analysed following the classical economics manner. Thirdly, an interactive aspect of a direct effect (reduction of the amount of waste) and an indirect effect (enhancement of quality of a secondary material)

¹I use DfE and eco-design interchangeably in this paper.

of EPR is explored, and fourthly, it is revealed that those mutual effects may lead to instability of an economy. Thus, we have a paradoxical result that an economy may possibly diverge from a long-run competitive equilibrium in which per capita consumption is maximized if direct and indirect DfE/eco-design mutually work strongly. This result is not obtained in any other studies.

In order to put the problem in an operational way by means of a Sraffian type of theoretical model, I interpret the above issue as follows: producers are assumed to bear financial EPR, so that they have to pay charges for disposal and/or recycling of an end-of-life product (ELP) which they have produced. Since they owe the costs of waste disposal and/or recycling of their products at a post-consumption stage, they take those costs into account when they design and produce products. However, the costs which producers bear may not be the ones which are obtained in a long-run competitive equilibrium, since product design, which affects production costs, cannot be adjusted by supply-demand balance; a market economy has to find out the correct costs which correspond to the ones obtained in a long-run competitive equilibrium. Starting from arbitrary product design and being lead by a price signal of waste treatment costs, can producers arrive at the correct product design which reflects a long-run competitive equilibrium? This is the problem which I try to solve by means of a dynamic economic model.

More specifically, I would like to solve the problem by exploring under which conditions a short-run competitive equilibrium converges to a long-run one in the neighbourhood of the latter equilibrium. This looks a typical stability problem which one often faces in a discussion of an ordinary general equilibrium model. However, there is a big difference between the present model and ordinary dynamic stability models; adjustment of economic variables is not made by supply-demand balance in the present model. Instead, I assume that an adjustment of choice of technique is made by profitability; choice of technique made by producers may not be optimal at an arbitrary moment since it is assumed to take time for an economy to find out the correct variables of DfE.

Related to this, there is another point which I would like to emphasize; I assume that supply-demand balance is always maintained in a short-run, although the position is different from a long-run competitive equilibrium. Hence, the adjustment process may seem to be similar to a *non-tatonnement* rather than *tatonnement*². Yet, there is an essential difference between the dynamics in the present paper and the one in the ordinary equilibrium analysis; in the present model, an adjustment movement occurs for pursuit of higher surplus (or value added), but such motivation is not found in the *non-tatonnement* argument.

There is one more important thing which I should mention in this context. It might be argued that the analytical aspect of the present paper seems to be close to that of the classical convergence theory, that is, the theory on convergence of the market price to the natural price. There have been many contributions in this field, which explore the dynamics of the capitalist economy in a very fruitful way (See for example Steedman 1984, Hosoda 1985, Dutt 1988, Duménil and Lévy 1991, Bellino 1997, Zhang 2006, Boggio 2008, Kiedrowski 2018, Bellino and Serrano 2018 and so on³). Certainly, the present paper shares a similar character to those studies, since both types of research ask how and on what conditions a short-run equilibrium converges to a long-run equilibrium. However, the present paper tries to solve a dynamic choice-of-technique problem under a uniform rate of profit and does not consider the movement process of capital goods which pursue a higher

²See Takayama (1974) on the concept of *tatonnement* and *non-tatonnement* processes.

³Only part of the contributions are cited here just for reference.

profit rate. Thus, the adjustment process is formulated quite differently from those studies although the spirit of the classical economic analysis is the same.

2 The basic model and assumptions

2.1 The production and consumption structure

The analytical model I adopt in this paper is basically the same as the one in Hosoda (2019). However, based upon the static model, I develop a dynamic equilibrium model and examine stability conditions in the neighbourhood of a long-run competitive equilibrium.

Following Hosoda (2019), let me start the explanation of an economy from the production side. I assume that there are four sectors in an economy; the first sector, which has only one production process, inputs its own product (a capital commodity) and labour, producing a capital commodity. The second sector is a consumption-commodity production sector, which inputs a capital commodity and labour, producing a consumption commodity. Input coefficients are assumed to depend upon a *discharge rate* of waste represented by θ_2 ($\in [0, 1]$) which expresses units (e.g. weight) of an ELP (end-of-life product) when this commodity turns to an ELP discharged after consumption. Thus, those coefficients are designated as $(a_{12}(\theta_2), l_2(\theta_2))$. I assume that larger value of the coefficients corresponds to smaller discharge rate θ_2 , since it is more costly and requires more inputs to reduce the amount of waste after consumption. I will explain the discharge rate in more detail later.

The third sector is the one which inputs a capital commodity, labour and an end-of-life product (ELP) which is discharged by households, producing a secondary material. This sector is considered a recycling sector, and a process which belongs to this sector is identified by the quality of a secondary material expressed by ρ ($\in [0, 1]$). Thus, the coefficients are designated as $(a_{13}(\rho), l_3(\rho), a_{33})$. I assume that larger value of the coefficients corresponds to larger ρ , since it is more costly and requires more inputs to produce a secondary material with higher quality. I assume that a_{33} is independent of ρ , since an effect of ρ on inputs of an ELP is ambiguous, so that it is assumed constant for simplicity. I will refer to a possibility of relaxation of this assumption in Appendix C.

The fourth sector is a consumption-commodity production sector. However, this sector is different from the second sector in the sense that it inputs a secondary material as well as a capital commodity and labour for producing a consumption commodity. The production condition of this sector is affected by the quality of a secondary material (ρ) and a discharge rate of waste (θ_4 ($\in [0, 1]$)) which expresses units (e.g. weight) of an ELP when this commodity turns to an ELP discharged after consumption. Hence, the coefficients are denoted as $(a_{14}(\rho, \theta_4), l_4(\rho, \theta_4), a_{44})$, where a_{44} is an input of a secondary material and assumed to be constant for the same reason as above. Inputs of a capital commodity and labour increase as the quality of the secondary material decreases and/or the discharge rate decreases, since these imply that the production becomes more costly. I will refer to the case in which a_{44} depends upon (ρ, θ_4) in Appendix C.

From the above explanation, it is understood that the first sector has only one production process while the other sectors have infinitely many processes, depending upon $(\rho, \theta_2, \theta_4)$. This implies that there is a problem of choice of technique in this economy, and that choice of technique is the problem of how $(\rho, \theta_2, \theta_4) \in [0, 1] \times [0, 1] \times [0, 1]$ is chosen. In a long-run competitive equilibrium, the cost-minimization principle is applicable to this problem, as Hosoda (2019) shows.

I assume that there is no consumption on the producers' side. They spend profit income only for investment, namely, purchase of a capital commodity for the next production.

Now, I can demonstrate the inter-sectoral relationship which I have just described above as follows:

Table 1: A structure of production sectors

	a capital commodity	a consumption commodity	ELP	a secondary material	labour		a capital commodity	a consumption commodity	secondary material
I	a_{11}	0	0	0	l_1	\rightarrow	1	0	0
II	$a_{12}(\theta_2)$	0	0	0	$l_2(\theta_2)$	\rightarrow	0	1	0
III	$a_{13}(\rho)$	0	a_{33}	0	$l_3(\rho)$	\rightarrow	0	0	1
IV	$a_{14}(\rho, \theta_4)$	0	0	a_{44}	$l_4(\rho, \theta_4)$	\rightarrow	0	1	0

Next, let me refer to households activities. Households are assumed to consume a consumption commodity produced by the second and fourth sectors. A consumption commodity which is produced by the second sector and the one produced by the fourth sector are not differentiated and are regarded as the same by households. They are not supposed to save and spend all their income for consumption.

They dispose of an ELP at the post-consumption stage. Since I assume that financial EPR is imposed on producers, households do not have to pay for treatment of waste disposal, and, instead, producers are supposed to owe the costs of waste treatment after consumption. A unit of consumption of the commodity produced by the second and fourth sectors produces θ_2 and θ_4 units of an ELP respectively. It should be noted that θ_2 may be equal to or different from θ_4 . Households are not concerned with the difference, if any, between the two discharge rates θ_2 and θ_4 , since whether a consumption commodity is produced by the second sector or the fourth sector is assumed to be indistinguishable firstly, and households are supposed not to have to pay for waste treatment secondly.

Then, a structure of creation of an ELP is described as follows:

Table 2: A structure of discharge of an ELP

a consumption commodity produced by process II	a consumption commodity produced by process IV		discharge of an ELP
1	-	\rightarrow	θ_2
-	1	\rightarrow	θ_4

Since producers do not consume and households do not save, the growth rate (g) equals to the profit rate (r). I assume that this equality always holds in a short-run equilibrium, defined as a sub-equilibrium later. I also assume that a profit rate, one of the distribution variables, is given for

an operational reason. However, the following argument is completely valid if I assume the other distribution variable, namely a wage rate, is given.

It is worth remarking on what is implied for the nature of ELP by introduction of financial EPR: ELP can be regarded as a joint product of a production process. It is clear to see this if Table 1 and 2 are combined together. One thing which is different from an ordinary joint product is that ELP could be negatively priced, so that producers are supposed to owe the treatment costs of ELP after consumption, when ELP is *bads* or *dis-commodity*.

2.2 Assumptions on production coefficients

Having shown the fundamental structure of production, consumption, and discharge of an ELP, I would like to give formal assumptions on the coefficients defined above⁴.

Assumption 1 (i) $\left(\frac{da_{12}(\theta_2)}{d\theta_2}, \frac{dl_2(\theta_2)}{d\theta_2}\right) \ll 0$ and $\left(\frac{\partial a_{14}(\rho, \theta_4)}{\partial \theta_4}, \frac{\partial l_4(\rho, \theta_4)}{\partial \theta_4}\right) \ll 0$,
(ii) $\left(\frac{da_{13}(\rho)}{d\rho}, \frac{dl_3(\rho)}{d\rho}\right) \gg 0$ and $\left(\frac{\partial a_{14}(\rho, \theta_4)}{\partial \rho}, \frac{\partial l_4(\rho, \theta_4)}{\partial \rho}\right) \ll 0$.

Assumption 1 (i) means that inputs into processes of consumption commodity production increase as discharge rates of an ELP decrease, since reduction of the amount of an ELP requires more inputs, meaning that production becomes more costly. Otherwise, a waste disposal problem never happens. The first part of Assumption 1 (ii) means that an increase in the quality of a secondary material requires more inputs for the recycling sector, since it needs more capital- and labour-intensive activities for the sector to enhance the quality of a secondary material. On the other hand, the second part of Assumption 1 (ii) means that an increase in the quality of a secondary material means less inputs in a consumption commodity production sector which inputs the secondary material, since inputs of a high-quality secondary material can contribute to savings of other inputs.

Assumption 2 (i) $\left(\frac{d}{d\theta_2} \left(\frac{da_{12}(\theta_2)}{d\theta_2}\right), \frac{d}{d\theta_2} \left(\frac{dl_2(\theta_2)}{d\theta_2}\right)\right) \gg 0$ and $\left(\frac{\partial}{\partial \theta_4} \left(\frac{\partial a_{14}(\rho, \theta_4)}{\partial \theta_4}\right), \frac{\partial}{\partial \theta_4} \left(\frac{\partial l_4(\rho, \theta_4)}{\partial \theta_4}\right)\right) \gg 0$
(ii) $\left(\frac{d}{d\rho} \left(\frac{da_{13}(\rho)}{d\rho}\right), \frac{d}{d\rho} \left(\frac{dl_3(\rho)}{d\rho}\right)\right) \gg 0$ and $\left(\frac{\partial}{\partial \rho} \left(\frac{\partial a_{14}(\rho, \theta_4)}{\partial \rho}\right), \frac{\partial}{\partial \rho} \left(\frac{\partial l_4(\rho, \theta_4)}{\partial \rho}\right)\right) \gg 0$.

Assumption 2 (i) implies that input coefficients of consumption-commodity production sectors II and IV increase more than proportionately as discharge rates decrease. Thus, production of a consumption commodity becomes more and more costly as producers try to reduce discharge rates, whether the commodity is produced by sector II or IV. The first part of Assumption 2 (ii) tells us that input coefficients of a recycling sector increase more than proportionately as the quality of a secondary material is increased. This means that recycling activities become more and more costly as recyclers try to enhance the quality of a secondary material. On the other hand, the second part of Assumption 2 (ii) tells us that input coefficients of consumption-commodity production sector IV decrease less than proportionately, meaning that the effect of the enhancement of quality of a secondary material on reduction of input coefficients becomes weaker as the quality becomes higher.

⁴The following assumptions are fundamentally the same as those adopted in Hosoda (2019).

The next assumption, which seems a little complicated, is on the *cross-effect* of a discharge rate and quality of a secondary material on input coefficients of a consumption-commodity production sector IV.

Assumption 3

$$\left(\frac{\partial}{\partial \rho} \left(\frac{\partial a_{14}(\rho, \theta_4)}{\partial \theta_4} \right), \frac{\partial}{\partial \rho} \left(\frac{\partial l_4(\rho, \theta_4)}{\partial \theta_4} \right) \right) \equiv \left(\frac{\partial}{\partial \theta_4} \left(\frac{\partial a_{14}(\rho, \theta_4)}{\partial \rho} \right), \frac{\partial}{\partial \theta_4} \left(\frac{\partial l_4(\rho, \theta_4)}{\partial \rho} \right) \right) \gg 0.$$

Let me explain the above assumption. Notice that $\partial a_{14}(\rho, \theta_4)/\partial \theta_4$ and $\partial l_4(\rho, \theta_4)/\partial \theta_4$ show how the input coefficients increase as discharge rate θ_4 decreases. Hence, those derivatives express an adverse effect of a direct DfE (i.e., a decrease of θ_4) on the production conditions of the fourth sector.

The assumption implies that this adverse effect is weakened by an increase in quality of a secondary material (ρ). Such an effect is often found in a real recycling field. Reduction of the amount of waste discharge goes hand in hand with enhancement of quality of a secondary material.

This is because enhancement of quality of a secondary material is supposed to work in a supportive manner to producers' efforts at the design stage for reducing waste discharge. For example, as the number of parts and materials in a product decreases, the amount of waste after consumption often decreases and the quality of a secondary material which is transformed from an ELP is enhanced at the same time⁵.

The same thing could be explained from a different angle. Derivatives $\partial a_{14}(\rho, \theta_4)/\partial \rho$ and $\partial l_4(\rho, \theta_4)/\partial \rho$ show how input coefficients decrease as quality of a secondary material ρ increases. This is nothing but an advantageous effect of the enhancement of quality of a secondary material on the production condition of sector IV. Yet, this positive effect is weakened as waste discharge rate θ_4 increases. It is often seen that quality of a secondary material cannot be raised in a circumstance in which producers do not care about the amount of waste discharged at the post-consumption stage.

Now, defining $a_{13}^+ \equiv (1+r)a_{13}(\rho)a_{44} + a_{14}(\rho, \theta_4)$ and $l_3^+ \equiv (1+r)l_3(\rho)a_{44} + l_4(\rho, \theta_4)$, I have

$$\left(\frac{\partial}{\partial \rho} \left(\frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} \right), \frac{\partial}{\partial \rho} \left(\frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \right) \right) \equiv \left(\frac{\partial}{\partial \theta_4} \left(\frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} \right), \frac{\partial}{\partial \theta_4} \left(\frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \right) \right) \gg 0. \quad (1)$$

Due to Assumption 2 (ii), I also have the following:

$$\left(\frac{\partial}{\partial \rho} \left(\frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} \right), \frac{\partial}{\partial \rho} \left(\frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \right) \right) \gg 0. \quad (2)$$

Finally in this subsection, I would like to adopt the following assumption:

Assumption 4 $(1+g)a_{33}a_{44} - \theta_i \equiv (1+g)a_{33}^+ - \theta_i > 0$ for $i = 2, 4$.

This assumption implies that recycling cannot produce more than a unit of a consumption commodity by inputting ELP created by consumption of a unit of the commodity. Due to this assumption, one can avoid a complicated situation caused by so-called up-cycling.

⁵This is not always the case, and we can find counterexamples.

2.3 Definition of an equilibrium

In this subsection, I would like to demonstrate concepts of an equilibrium, upon which I develop a dynamic model later. Although the present paper is fundamentally based upon Hosoda (2019), I pick up a special case explained in that paper; a case in which an ELP is *bads* (*dis-commodity*) and producers are financially responsible for treatment of an ELP, possibly due to extended producer responsibility (EPR)⁶.

A cost-price equation in this economy is described as follows:

$$\begin{cases} (1+r)p_1a_{11} + wl_1 &= p_1 \\ (1+r)p_1a_{12}(\theta_2) + wl_2(\theta_2) &= p_2 + p_3\theta_2 \\ (1+r)\{p_1a_{13}(\rho) + p_3a_{33}\} + wl_3(\rho) &= p_{4\rho} \\ (1+r)\{p_1a_{14}(\rho, \theta_4) + p_{4\rho}a_{44}\} + wl_4(\rho, \theta_4) &= p_2 + p_3\theta_4, \end{cases} \quad (3)$$

where p_3 is negative.

Since producers are supposed to be responsible financially for treatment of an ELP, the negative value of an ELP must be taken into account as costs by producers, so that $p_3\theta_2$ and $p_3\theta_4$ appear on the right-hand side of the second and fourth equations respectively. It is easy to see that the nature of ELP as a joint product is expressed in (3).

Needless to say, not all $(p_1, p_2, p_3, p_{4\rho}, \rho, \theta_2, \theta_4)$ which satisfy the above equations are long-run competitive equilibrium solutions. It must be noted that each process in sector II, III and IV is completely identified with θ_2 , ρ and (θ_4, ρ) respectively, so that how to choose $(\rho, \theta_2, \theta_4)$ is nothing but a matter of choice of technique.

For the purpose of choice of technique, it may be convenient for us to transform (3) into a reduced form by means of vertical integration⁷. Let me integrate the third and fourth sectors for the same (ρ, θ_4) as follows:

$$(1+r)\{p_1a_{13}^+(\rho, \theta_4) + p_3a_{33}^+\} + wl_3^+(\rho, \theta_4) = p_2 + p_3\theta_4,$$

where $a_{13}^+(\rho, \theta_4) \equiv (1+r)a_{13}(\rho)a_{44} + a_{14}(\rho, \theta_4)$, $a_{33}^+ \equiv (1+r)a_{33}a_{44}$ and $l_3^+(\rho, \theta_4) \equiv (1+r)a_{44}l_3(\rho) + l_4(\rho, \theta_4)$.

Then, I can transform (3) into the following:

$$\begin{cases} (1+r)p_1a_{11} + wl_1 &= p_1 \\ (1+r)p_1a_{12}(\theta_2) + wl_2(\theta_2) &= p_2 + p_3\theta_2, \\ (1+r)\{p_1a_{13}^+(\rho, \theta_4) + p_3a_{33}^+\} + wl_3^+(\rho, \theta_4) &= p_2 + p_3\theta_4 \end{cases} \quad (4)$$

where (p_1, p_2, p_3, w) is determined once $(\rho, \theta_2, \theta_4)$ and *numeraire* are determined. I adopt a consumption commodity as *numeraire*, and put $p_2 = 1$.

Let me call a solution obtained by (4) for given $(\rho, \theta_2, \theta_4)$ a *sub-equilibrium solution* corresponding to $(\rho, \theta_2, \theta_4)$. It is a candidate for a *long-run competitive equilibrium solution*, but may not coincide with that solution.

⁶In Hosoda (2019), a condition under which an ELP becomes *bads* is stated explicitly.

⁷The notion of vertical integration here is different from that of Pasinetti (1977).

Since, in a long-run competitive equilibrium, a price vector $(p_1^*, 1, p_3^*, w^*)$ corresponding to an equilibrium DfE parameter vector $(\rho^*, \theta_2^*, \theta_4^*)$ should not make any extra profit for $(\rho, \theta_2, \theta_4) \in [0, 1] \times [0, 1] \times [0, 1]$, the following must hold in an equilibrium:

$$\begin{cases} (1+r)p_1^*a_{11} + w^*l_1 &= p_1^* \\ (1+r)p_1^*a_{12}(\theta_2^*) + w^*l_2(\theta_2^*) &= 1 + p_3^*\theta_2^* \\ (1+r)\{p_1^*a_{13}^+(\rho^*, \theta_4^*) + p_3^*a_{33}^+\} + w^*l_3^+(\rho^*, \theta_4^*) &= 1 + p_3^*\theta_4^*, \end{cases} \quad (5)$$

and

$$\begin{cases} (1+r)p_1^*a_{11} + w^*l_1 &= p_1^* \\ (1+r)p_1^*a_{12}(\theta_2) + w^*l_2(\theta_2) &\geq 1 + p_3^*\theta_2 \\ (1+r)\{p_1^*a_{13}^+(\rho, \theta_4) + p_3^*a_{33}^+\} + w^*l_3^+(\rho, \theta_4) &\geq 1 + p_3^*\theta_4 \end{cases} \quad (6)$$

for any $(\rho, \theta_2, \theta_4) \in [0, 1] \times [0, 1] \times [0, 1]$. A long-run competitive equilibrium solution $(p_1^*, 1, p_3^*, w^*, \rho^*, \theta_2^*, \theta_4^*)$ consists of a price formation part $(p_1^*, 1, p_3^*, w^*)$ and a technique formation part $(\rho^*, \theta_2^*, \theta_4^*)$. A long-run competitive equilibrium price vector $(p_1^*, 1, p_3^*, w^*)$ guarantees profits with a unique rate to each process in the technique represented by $(\rho^*, \theta_2^*, \theta_4^*)$, while it cannot create any extra profits for processes in any other technique represented by $(\rho, \theta_2, \theta_4) \neq (\rho^*, \theta_2^*, \theta_4^*)$, as I have already mentioned.

The existence of a long-run equilibrium solution is proved by Hosoda (2019). I assume that the equilibrium solution is interior, i.e., $(\theta_2^*, \rho^*, \theta_4^*) \in (0, 1) \times (0, 1) \times (0, 1)$, for simplicity.

It is worth mentioning that producers may not be able to find instantaneously a proper vector $(\rho^*, \theta_2^*, \theta_4^*)$ corresponding to a long-run competitive equilibrium when EPR is introduced and producers are required to pay for treatment of an ELP. Clearly, a long-run equilibrium solution is the best for both producers and households, since the cost-minimization principle is fulfilled and per capita consumption is maximized in a long-run competitive equilibrium⁸. Then, can producers arrive at a long-run competitive equilibrium position, starting from an arbitrarily given $(\rho, \theta_2, \theta_4)$, only by means of a price signal of waste treatment? This is the fundamental problem I try to solve in this paper.

Finally, in this subsection, let me mention a quantity side. In a long-run competitive equilibrium, the following supply-demand conditions are satisfied (Hosoda 2019):

$$\begin{cases} (1+g)a_{11}x_1 + a_{12}(\theta_2)x_2 + a_{13}(\rho)x_3 + a_{14}(\rho, \theta_4)x_4 &= x_1 \\ c &= x_2 + x_4 \\ (1+g)a_{33}x_{3\rho} &= \theta_2x_2 + \theta_4x_4 \\ (1+g)a_{44}x_4 &= x_{3\rho} \\ l_1x_1 + l_2(\theta_2)x_2 + l_3(\rho)x_3 + l_4(\rho, \theta_4)x_4 &= 1, \end{cases} \quad (7)$$

where $x_i (i = 1, 2, 3\rho, 4)$ and c are an activity level (an output level) of each process and per capita consumption respectively. It must be noted that the production process of a secondary material (the third process) is characterized by its quality denoted by ρ . Thus, ρ is attached to the subscript of the third activity level.

The first and second equations show the supply-demand balance of a capital-commodity and a consumption-commodity respectively. As for a consumption commodity, it does not matter whether it may be produced by the second or fourth sector.

⁸This is proved by Hosoda (2019).

The third equation means the supply-demand balance of an ELP, which is discharged by households and treated by the third sector, namely the recycling sector. This sector produces a secondary material, which is demanded by the fourth sector, and the fourth equation shows the supply-demand balance of a secondary material.

The last equation is nothing but the supply-demand equality of the labour force. The total supply of labour is normalized as unity.

If $(\rho, \theta_2, \theta_4)$ determined by (5) is applied to (7), then $(x_1^*, x_2^*, x_{3\rho}^*, x_4^*)$ which satisfies (7) is an equilibrium solution for a quantity system.⁹

3 An adjustment process and stability (1)

Based upon (3), (5) and (6), I formulate an adjustment process of a DfE parameter vector $(\rho, \theta_2, \theta_4)$ in the neighbourhood of $(\rho^*, \theta_2^*, \theta_4^*)$. Let us assume $(\rho, \theta_2, \theta_4)$ is given arbitrarily in the neighbourhood of $(\rho^*, \theta_2^*, \theta_4^*)$ as a starting point, and consider how a short-run equilibrium is adjusted. It must be remembered that a short-run equilibrium solution is nothing but a sub-equilibrium one given by (4).

3.1 Adjustment of θ_2

First, let me consider the adjustment of θ_2 . If

$$(1+r)p_1a_{12}(\theta_2 + \Delta\theta_2) + wl_2(\theta_2 + \Delta\theta_2) < 1 + p_3(\theta_2 + \Delta\theta_2) \quad (8)$$

holds for $\Delta\theta_2$ (where $\Delta\theta_2 \leq 0$ and $|\theta_2|$ is sufficiently small), there is motivation for producers to change θ_2 , since there is a chance for obtaining more surplus (a higher wage rate when a profit rate is given or a higher profit rate when a wage rate is given).

If the above inequality (8) holds for positive $\Delta\theta_2$, then there is an increase in θ_2 . If it holds for negative θ_2 , there is a decrease in θ_2 . With the help of the second equation of (4), inequality (8) is transformed into

$$\begin{cases} p_3 - \frac{(1+r)p_1 \{a_{12}(\theta_2 + \Delta\theta_2) - a_{12}(\theta_2)\} + w \{l_2(\theta_2 + \Delta\theta_2) - l_2(\theta_2)\}}{\Delta\theta_2} > 0 : \Delta\theta_2 > 0 \\ p_3 - \frac{(1+r)p_1 \{a_{12}(\theta_2 + \Delta\theta_2) - a_{12}(\theta_2)\} + w \{l_2(\theta_2 + \Delta\theta_2) - l_2(\theta_2)\}}{\Delta\theta_2} < 0 : \Delta\theta_2 < 0 \end{cases} \quad (9)$$

Considering (9), I may formulate the adjustment process of θ_2 as

$$\frac{\Delta\theta_2}{\Delta t} = \alpha \left[p_3 - \frac{(1+r)p_1 \{a_{12}(\theta_2 + \Delta\theta_2) - a_{12}(\theta_2)\} + w \{l_2(\theta_2 + \Delta\theta_2) - l_2(\theta_2)\}}{\Delta\theta_2} \right],$$

where t denotes time and α is a positive constant which shows an adjustment parameter. Let $\Delta t \rightarrow 0$ and $\Delta\theta_2 \rightarrow 0$, and then I can obtain

$$\frac{d\theta_2}{dt} = \alpha [p_3 - \{(1+r)p_1 a'_{12}(\theta_2) + w l'_2(\theta_2)\}]. \quad (10)$$

⁹See Hosoda (2019) on a quantity solution.

3.2 Adjustment of ρ

Let me examine how ρ is adjusted. The adjustment of ρ is not as simple as that of θ_2 , since the change in ρ affects the third and fourth sectors in an opposite way. If ρ increases (decreases), the third sector is negatively (positively) affected by cost increase (cost decrease), while the fourth sector is positively (negatively) affected by cost decrease (cost increase). Thus, there is conflict of interests between two sectors, so that the adjustment of ρ may not occur. However, I can show that there is a win-win transaction by transfer of the gains obtained by the adjustment of ρ , if

$$(1+r) \{p_1 a_{13}^+(\rho + \Delta\rho, \theta_4) + p_3 a_{33}^+\} + w l_3^+(\rho + \Delta\rho, \theta_4) < 1 + p_3 \theta_4 \quad (11)$$

holds for $\Delta\rho$ (where $\Delta\rho \leq 0$, and $|\Delta\rho|$ is sufficiently small). If so, there is motivation for producers to change ρ , since there is a chance for both the third and fourth sectors to obtain more surplus (a higher wage rate or a higher profit rate).

Lemma 1 *If (11) holds for $\Delta\rho$ (where $\Delta\rho \leq 0$, and $|\Delta\rho|$ is sufficiently small), both the third and fourth sectors could be better off by distribution of the gains which are obtained by the adjustment of ρ .*

Proof. See Appendix A. ■

According to the above lemma, there is a win-win chance for the third and fourth sectors as far as (11) holds, so that I assume there is adjustment for ρ in this section¹⁰. Certainly, there may be a case in which such adjustment does not occur in reality even if (11) holds, due to the difficulty of the transfer of the gains between two sectors. In such a case, ρ remains unadjusted. I deal with the case in the next section.

Therefore, I assume that the adjustment of ρ occurs when (11) holds for a while. Then, it is natural to assume that there is an increase in ρ if (11) holds for positive $\Delta\rho$. On the other hand, if it holds for negative $\Delta\rho$, there is a decrease in ρ .

Hence from inequality (11), I can formulate the adjustment process as follows:

$$\frac{\Delta\rho}{\Delta t} = -\beta \left[(1+r)p_1 \frac{\{a_{13}^+(\rho + \Delta\rho, \theta_4) - a_{13}^+(\rho, \theta_4)\}}{\Delta\rho} + w \frac{\{l_3^+(\rho + \Delta\rho, \theta_4) - l_3^+(\rho, \theta_4)\}}{\Delta\rho} \right],$$

where β is a positive constant which shows an adjustment parameter. Letting $\Delta t \rightarrow 0$ and $\Delta\rho \rightarrow 0$, I can obtain

$$\frac{d\rho}{dt} = -\beta \left[(1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \right]. \quad (12)$$

¹⁰If the adjustment of ρ occurs, the amount of the transfer payment matters. Yet, we do not have to discuss it since the qualitative nature of the adjustment is not affected by the amount.

3.3 Adjustment of θ_4

The adjustment of θ_4 is the same as that of θ_2 on the supposition that ρ is given. If

$$(1+r) \{p_1 a_{13}^+(\rho, \theta_4 + \Delta\theta_4) + p_3 a_{33}^+\} + w l_3^+(\rho, \theta_4 + \Delta\theta_4) < p_2 + p_3 (\theta_4 + \Delta\theta_4) \quad (13)$$

holds for $\Delta\theta_4$ (where $\Delta\theta_4 \leq 0$ and $|\theta_4|$ is sufficiently small), there is motivation for producers to change θ_4 , since there is a chance for obtaining more surplus (a higher wage rate or a higher profit rate).

If the above inequality (13) holds for positive $\Delta\theta_4$, then there is an increase in θ_4 . If it holds for negative θ_4 , there is a decrease in θ_4 . With the help of the third equation of (4), inequality (13) is transformed into

$$\begin{cases} p_3 - \frac{(1+r)p_1 \{a_{13}^+(\rho, \theta_4 + \Delta\theta_4) - a_{13}^+(\rho, \theta_4)\} + w \{l_3^+(\rho, \theta_4 + \Delta\theta_4) - l_3^+(\rho, \theta_4)\}}{\Delta\theta_4} > 0 : \Delta\theta_4 > 0 \\ p_3 - \frac{(1+r)p_1 \{a_{13}^+(\rho, \theta_4 + \Delta\theta_4) - a_{13}^+(\rho, \theta_4)\} + w \{l_3^+(\rho, \theta_4 + \Delta\theta_4) - l_3^+(\rho, \theta_4)\}}{\Delta\theta_4} < 0 : \Delta\theta_4 < 0 \end{cases} \quad (14)$$

Considering (14), I may formulate the adjustment process of θ_4 as

$$\frac{\Delta\theta_4}{\Delta t} = \gamma \left[p_3 - \frac{(1+r)p_1 \{a_{13}^+(\rho, \theta_4 + \Delta\theta_4) - a_{13}^+(\rho, \theta_4)\} + w \{l_3^+(\rho, \theta_4 + \Delta\theta_4) - l_3^+(\rho, \theta_4)\}}{\Delta\theta_4} \right],$$

where γ is a positive constant which shows an adjustment parameter. Let $\Delta t \rightarrow 0$ and $\Delta\theta_4 \rightarrow 0$, and then I can obtain

$$\frac{d\theta_4}{dt} = \gamma \left[p_3 - \left\{ (1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \right\} \right]. \quad (15)$$

Putting (10), (12) and (15) together, I can obtain the following:

$$\begin{cases} \frac{d\theta_2}{dt} \equiv \dot{\theta}_2 = \alpha [p_3 - \{(1+r)p_1 a'_{12}(\theta_2) + w l'_2(\theta_2)\}] \equiv \psi_1(\theta_2, \rho, \theta_4) \\ \frac{d\rho}{dt} \equiv \dot{\rho} = -\beta \left[(1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \right] \equiv \psi_2(\theta_2, \rho, \theta_4) \\ \frac{d\theta_4}{dt} \equiv \dot{\theta}_4 = \gamma \left[p_3 - \left\{ (1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \right\} \right] \equiv \psi_3(\theta_2, \rho, \theta_4). \end{cases} \quad (16)$$

3.4 Stability of the adjustment process

The purpose of this paper is to examine movement of $(\theta_2, \rho, \theta_4)$ expressed by (16) in the neighbourhood of a long-run equilibrium point represented by $(\theta_2^*, \rho^*, \theta_4^*)$. Notice that (p_1, p_3, w) is a function of $(\theta_2, \rho, \theta_4)$.

Let me demonstrate whether a long-run competitive equilibrium point is stable or not, and on what condition it is asymptotically stable. Let us confirm an important nature of a long-run competitive equilibrium as follows:

$$\begin{cases} p_3 - \{(1+r)p_1 a'_{12}(\theta_2)|_{\theta^*} + w l'_2(\theta_2)|_{\theta^*}\} = 0 \\ (1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} \Big|_{(\rho^*, \theta_4^*)} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \Big|_{(\rho^*, \theta_4^*)} = 0. \\ p_3 - \left\{ (1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} \Big|_{(\rho^*, \theta_4^*)} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \Big|_{(\rho^*, \theta_4^*)} \right\} = 0 \end{cases} \quad (17)$$

These equalities hold because there is no room for increases in profits in any sector once a long-run competitive equilibrium is attained and so $(\dot{\theta}_2, \dot{\rho}, \dot{\theta}_4) = 0$ holds.

Now, I can show the following important nature. Defining $\Phi(\theta_2, \rho, \theta_4)$ as

$$\Phi(\theta_2, \rho, \theta_4) = \begin{pmatrix} \frac{\partial p_1}{\partial \theta_2} & \frac{\partial p_3}{\partial \theta_2} & \frac{\partial w}{\partial \theta_2} \\ \frac{\partial p_1}{\partial \rho} & \frac{\partial p_3}{\partial \rho} & \frac{\partial w}{\partial \rho} \\ \frac{\partial p_1}{\partial \theta_4} & \frac{\partial p_3}{\partial \theta_4} & \frac{\partial w}{\partial \theta_4} \end{pmatrix}.$$

I have the very powerful lemma as follows:

Lemma 2 $\Phi(\theta_2, \rho, \theta_4) \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = 0$.

Proof. See Appendix B. ■

Thanks to Lemma 2, I can linearize (16) in the neighbourhood of a long-run competitive equilibrium and obtain the following simple differential equation system:

$$\begin{pmatrix} \dot{\theta}_2 \\ \dot{\rho} \\ \dot{\theta}_4 \end{pmatrix} = \Psi^* \begin{pmatrix} \theta_2 - \theta_2^* \\ \rho - \rho^* \\ \theta_4 - \theta_4^* \end{pmatrix}, \quad (18)$$

where Ψ^* is a Jacobian of the right-hand side of (16) and calculated as

$$\Psi^* \equiv \left(\begin{array}{ccc} \frac{\partial \psi_1}{\partial \theta_2} & \frac{\partial \psi_1}{\partial \rho} & \frac{\partial \psi_1}{\partial \theta_4} \\ \frac{\partial \psi_2}{\partial \theta_2} & \frac{\partial \psi_2}{\partial \rho} & \frac{\partial \psi_2}{\partial \theta_4} \\ \frac{\partial \psi_3}{\partial \theta_2} & \frac{\partial \psi_3}{\partial \rho} & \frac{\partial \psi_3}{\partial \theta_4} \end{array} \right) \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & B_2 & B_3 \\ 0 & C_2 & C_3 \end{pmatrix},$$

and

$$\begin{aligned}
A_1 &= -\alpha \{ (1+r)p_1 a''_{12}(\theta_2) + w l''_2(\theta_2) \} (< 0) \\
B_2 &= -\beta \left[(1+r)p_1 \frac{\partial}{\partial \rho} \left(\frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} \right) + w \frac{\partial}{\partial \rho} \left(\frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \right) \right] (< 0) \\
B_3 &= -\beta \left[(1+r)p_1 \frac{\partial}{\partial \theta_4} \left(\frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} \right) + w \frac{\partial}{\partial \theta_4} \left(\frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \right) \right] (< 0) \\
C_2 &= -\gamma \left[(1+r)p_1 \frac{\partial}{\partial \rho} \left(\frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} \right) + w \frac{\partial}{\partial \rho} \left(\frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \right) \right] (< 0) \\
C_3 &= -\gamma \left[(1+r)p_1 \frac{\partial}{\partial \theta_4} \left(\frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} \right) + w \frac{\partial}{\partial \theta_4} \left(\frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \right) \right] (< 0).
\end{aligned}$$

Since (16) is an autonomous differential equation system, its linear approximation (18) is *uniformly good approximation* to the original differential equation system, so that I can safely mention the nature of asymptotic stability in the neighbourhood of a long-run competitive equilibrium by means of (18)¹¹.

The characteristic equation is

$$|\lambda I - \Psi^*| = 0,$$

which is equivalent to

$$(\lambda - A_1) \{ \lambda^2 - (B_2 + C_3) \lambda + B_2 C_3 - B_3 C_2 \} = 0.$$

Clearly, $\lambda_1^* = A_1 (< 0)$ holds, so that θ_2 converges to θ_2^* starting from any initial point near θ_2^* . The signs of the other characteristic roots, λ_2 and λ_3 , depend upon

$$\text{trace } \Psi^{*(-)} \equiv B_2 + C_3 < 0 \quad \det \Psi^{*(-)} \equiv B_2 C_3 - B_3 C_2 \lesseqgtr 0,$$

where $\Psi^{*(-)}$ is defined as

$$\Psi^{*(-)} \equiv \begin{pmatrix} B_2 & B_3 \\ C_2 & C_3 \end{pmatrix}.$$

If $\det \Psi^{*(-)}$ is positive, both λ_2^* and λ_3^* are negative since $\text{trace } \Psi^{*(-)}$ is negative, so that (ρ, θ_4) converges to (ρ^*, θ_4^*) starting from any initial point. Consequently, the long-run competitive equilibrium is locally asymptotically stable.

On the other hand, if $\det \Psi^{*(-)}$ is negative, either λ_2^* or λ_3^* is positive, so that (ρ, θ_4) does not converge to (ρ^*, θ_4^*) , and diverges from (ρ^*, θ_4^*) unless the initial point exists on a very specific trajectory. Since such a thing happens only with rare exceptions, I may safely say that the long-run competitive equilibrium is unstable.

Now we have the following proposition:

Proposition 1 *If and only if $B_2 C_3 > B_3 C_2$ holds, the long-run competitive equilibrium point is locally asymptotically stable. Otherwise, it is unstable.*

¹¹See Gandolfo (2009) for the detailed explanation on *uniformly good approximation*.

Let us consider the stability condition. The terms on the left-hand side of the inequality, namely B_2 and C_3 , express the effects of changes of ρ and θ_4 on the changes of production costs of the vertically integrated recycling processes, whereas those on the right-hand side, namely B_3 and C_2 , express the cross effects (or indirect effects) of the changes of ρ and θ_4 on the changes of the production costs. Thus, intuitively speaking, B_2C_3 expresses the degree of cost increase by enhancement of eco-design (an increase in ρ and/or a decrease in θ_4), while B_3C_2 expresses cross effects of cost decrease by enhancement of eco-design. If the former effects surpass the latter, stability follows, but otherwise, instability follows.

I would like to explain intuitively how the cross effects work against stability. Suppose that $\rho < \rho^*$ and $\theta_4 > \theta_4^*$ initially. Clearly, an increase in ρ and a decrease in θ_4 toward (ρ^*, θ_4^*) increase surplus which could be distributed as wages or profits. So, the adjustment might seem to be made in this direction. However, the cross effects give a counter effect to such changes; they give motivation to producers to decrease ρ and increase θ_4 , since cost decreases caused by such changes may possibly be larger so that those effects surpass the cost-decrease effects caused by an increase in ρ and a decrease in θ_4 .

This result demonstrates a *paradox of eco-design*. If the cross effects of enhancement of quality of a secondary material and reduction of waste discharge on input coefficients are large, a short-run equilibrium (sub-equilibrium) does not converge to a long-run competitive equilibrium, and diverges from it. Thus, financial EPR neither leads an economy to the position of a long-run competitive equilibrium which maximizes per capita consumption nor enhances DfE or eco-design.

It is paradoxical because one of the purposes of EPR is to promote DfE or eco-design (OECD 2001) but this purpose cannot be fulfilled under a certain condition, due to the cross effects of DfE or eco-design. For the financial responsibility of EPR to be effective, the cross effects must be small. Presumably there are some cases in which cross effects are small or even negative for some commodities. The authority should be careful if he or she tries to apply financial EPR to products, and is required to examine whether the cross effects are large or small.

4 An Adjustment Process and Stability (2)

In the previous section, I have assumed that adjustment of ρ is made when (11) holds, since there is a possibility that both the third and fourth sectors can be better off by the adjustment if there is proper transfer of the gains between the two sectors, thanks to Lemma 1. Yet, such adjustment would not occur in reality possibly, due to failure of the negotiation of both sectors on transfer payment. Then, ρ may be fixed at arbitrary value ($\rho = \bar{\rho}$). Let me analyse this case.

4.1 Adjustment of θ_2 and θ_4

When ρ is fixed, i.e., $\rho = \bar{\rho}$, arbitrarily, a sub-equilibrium solution corresponding to $\rho = \bar{\rho}$ which maximizes per capita consumption is expressed as $(\theta_2^*, \bar{\rho}, \theta_4^{**})$, where $\theta_4^{**} = \theta_4(\bar{\rho})$. This solution is obtained by putting $\rho = \bar{\rho}$ into equations (5) and (6).

Since adjustment is made only by changes in θ_2 and θ_4 , I can deduce a dynamic adjustment process by the same argument as in the previous section as follows:

$$\begin{cases} \frac{d\theta_2}{dt} \equiv \dot{\theta}_2 = \alpha [p_3 - \{(1+r)a'_{12}(\theta_2) + wl_2(\theta_2)\}] \equiv \psi_1(\theta_2, \theta_4) \\ \frac{d\theta_4}{dt} \equiv \dot{\theta}_4 = \gamma \left[p_3 - \left\{ (1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \right\} \right] \equiv \psi_3(\theta_2, \theta_4). \end{cases} \quad (19)$$

Notice that (p_1, p_3, w) is a function of θ_2 and θ_4 .

4.2 Stability of an adjustment process

Linear approximation of (19) in the neighbourhood of $(\theta_2^*, \theta_4^{**})$ is expressed as follows:

$$\begin{pmatrix} \dot{\theta}_2 \\ \dot{\theta}_4 \end{pmatrix} = \Psi_{(-)}^* \begin{pmatrix} \theta_2 - \theta_2^* \\ \theta_4 - \theta_4^{**} \end{pmatrix}, \quad (20)$$

where $\Psi_{(-)}$ is defined and calculated as

$$\Psi_{(-)}^* \equiv \left(\begin{array}{cc} \frac{\partial \psi_1}{\partial \theta_2} & \frac{\partial \psi_1}{\partial \theta_4} \\ \frac{\partial \psi_3}{\partial \theta_2} & \frac{\partial \psi_3}{\partial \theta_4} \end{array} \right) \bigg|_{(\theta_2^*, \theta_4^{**})} = \begin{pmatrix} A_1 & 0 \\ 0 & C_3 \end{pmatrix},$$

since the result corresponding to Lemma 2 in the case where ρ is constant holds valid in the above system so that (p_1, p_3, w) is invariant to θ_2 and θ_4 at $(\theta_2^*, \theta_4^{**})$.

The characteristic equation is expressed as

$$|\lambda I - \Psi_{(-)}^*| = 0$$

from which

$$\lambda_1 = A_1 < 0 \text{ and } \lambda_2 = C_3 < 0.$$

This implies that a sub-equilibrium point obtained under the assumption $\rho = \bar{\rho}$ is locally asymptotically stable. Thus, starting from an arbitrary point in the neighbourhood of $(\theta_2^*, \theta_4^{**})$ where $\theta_4^{**} = \theta_4(\bar{\rho})$, a point (θ_2, θ_4) converges to $(\theta_2^*, \theta_4^{**})$. Thus, I have obtained the following result.

Proposition 2 *Suppose ρ is given. Then, a sub-equilibrium point $(\theta_2^*, \theta_4^{**})$ of (20) is locally asymptotically stable.*

This result could be anticipated from Proposition 1, since the dynamic process in this section may be regarded as a special case of the one formulated in the previous section. Since ρ is given in the model of this section, there is no cross effects of quality of a secondary material and a discharge rate. Proposition 2 strengthens the main idea of this paper: if we are allowed to consider the effects of financial EPR only on reduction of ELP which is bads or dis-commodity, we can safely state that it works very well in the sense that the adjustment process induced by financial EPR is stable. Yet, it must be noticed that the stable sub-equilibrium obtained under the assumption $\rho = \bar{\rho}$ is different from the long-run equilibrium which maximizes per capita consumption under the assumption that ρ is a variable.

5 Concluding remarks

By means of a Sraffian type of economic model, I have examined whether financial EPR can work well for enhancement of DfE or eco-design. For this purpose, I have examined how a competitive economy which pursues more surplus changes waste discharge rates and quality of a secondary material, and formulated a dynamic model which describes the adjustment process.

Contrary to the traditional argument (OECD 2001, 2016), which emphasizes the effectiveness of financial EPR, I have demonstrated that financial EPR can attain the target of enhancement of DfE/eco-design only under a specific condition; only when cross effects of reduction of a waste discharge rate and enhancement of quality of a secondary material on input coefficients in the integrated recycling sectors are small, a stability of the adjustment process follows and financial EPR can attain the target. Otherwise, such a stability result cannot be obtained, so that financial EPR cannot promote proper DfE/eco-design.

I have also considered the case in which quality of a secondary material is not adjusted somehow, for instance, possibly by institutional failure of transfer of gains which may be obtained by the adjustment of quality of a secondary material. In this case, a sub-equilibrium point is stable in its neighbourhood, although this equilibrium is not a long-run competitive one except by a fluke, so that per capita consumption is not maximized.

Finally, I would like to mention an aspect of income distribution in the adjustment process. I have assumed that adjustment of discharge rates and quality is made as far as there is a possibility of an increase in surplus, but, at the same time, that a uniform rate of profit is given and prevails in each sub-equilibrium. This implies that a wage rate, and so per capita consumption, increase in the process of adjustment¹². If a long-run competitive equilibrium is stable, a wage rate, and so per capita consumption, always increase and get closer and closer to the maximum value. Conversely, if a uniform wage rate is assumed to be given and prevails in each sub-equilibrium, a profit rate increases and is finally maximized in a stable economy. On the contrary, if the stability condition is not satisfied, a wage rate, and so per capita consumption, diverge from the maximum value or a profit rate diverges from the maximum value.

The price signal of waste in financial EPR is sometimes too much emphasized. This should be so, because one tends to consider an analogy of the price signal of an ordinary commodity, which has parameter function for balancing supply and demand. It has been proved in this paper that such an analogy cannot be applicable to the adjustment of DfE/eco-design. The authority should be prudent when he/she is required to apply financial EPR to some products. Otherwise, the intended target cannot be attained and an economy cannot be stabilized.

¹²This can be shown easily by a parallel argument to the conventional theory of choice of technique.

A Appendix

Proof of Lemma 1.

Suppose (3) and (11) hold for $\Delta\rho > 0$. Define A , B , C and D as

$$\begin{cases} A \equiv (1+r) \{p_1 a_{13}(\rho + \Delta\rho) + p_3 a_{33}\} + w l_3(\rho + \Delta\rho) \\ B \equiv p_{4\rho} \\ C \equiv (1+r) \{p_1 a_{14}(\rho + \Delta\rho, \theta_4) + p_{4\rho} a_{44}\} + w l_4(\rho + \Delta\rho, \theta_4) \\ D \equiv 1 + p_3 \theta_4 \end{cases}.$$

Then, the following holds:

$$\begin{aligned} (11) \quad &\Leftrightarrow C + (1+r)a_{44}A - (1+r)a_{44}B < D \\ &\Leftrightarrow \exists \tau > 0 \text{ s.t. } (1+r)a_{44}(A - B) < \tau < D - C \\ &\Leftrightarrow \exists \tau > 0 \text{ s.t. } A - \frac{\tau}{(1+r)a_{44}} < B \text{ and } C + \tau < D \end{aligned}.$$

By definition of A , B , C and D , we know that the adjustment $\Delta\rho$ makes both the third and fourth sectors better off by transfer of the gains by the adjustment from the fourth sector to the third sector.

Suppose (3) and (11) hold for $\Delta\rho < 0$. Then

$$\begin{aligned} (11) \quad &\Leftrightarrow C + (1+r)a_{44}A - (1+r)a_{44}B < D \\ &\Leftrightarrow \exists \tau > 0 \text{ s.t. } (1+r)a_{44}(B - A) > \tau > C - D \\ &\Leftrightarrow \exists \tau > 0 \text{ s.t. } A + \frac{\tau}{(1+r)a_{44}} < B \text{ and } C - \tau < D \end{aligned}.$$

For the same reason above, we know that the adjustment $\Delta\rho$ makes both the third and fourth sectors better off by transfer of the gains by the adjustment from the third sector to the fourth sector. ■

B Appendix

Proof of Lemma 2.

It is easy to calculate

$$\frac{\partial w}{\partial \theta_2} = \frac{\{1 - (1+r)a_{11}\}}{l_1} \frac{\partial p_1}{\partial \theta_2}, \quad (21)$$

$$-\frac{\partial p_3}{\partial \theta_2} = \frac{(1+r)a_{13}^+(\rho, \theta_4)l_1 + l_3^+(\rho, \theta_4)\{1 - (1+r)a_{11}\}}{l_1 \{(1+r)a_{33}^+ - \theta_4\}} \frac{\partial p_1}{\partial \theta_2} \quad (22)$$

and

$$\left[\frac{(1+r)a_{12}(\theta_2)l_1 + l_2(\theta_2)\{1 - (1+r)a_{11}\}}{l_1} + \theta_2 \frac{(1+r)a_{13}^+(\rho, \theta_4)l_1 + l_3^+(\rho, \theta_4)\{1 - (1+r)a_{11}\}}{l_1\{(1+r)a_{33}^+ - \theta_4\}} \right] \frac{\partial p_1}{\partial \theta_2} = p_3 - \{(1+r)a'_{12}(\theta_2) + wl'_2(\theta_2)\}. \quad (23)$$

Since the right-hand side of (23) is zero at $(\theta_2^*, \rho^*, \theta_4^*)$ thanks to (17) and $(1+r)a_{33}^+ - \theta_4 > 0$ due to Assumption 4, considering (21) and (22), we know the following holds:

$$0 = \frac{\partial p_1}{\partial \theta_2} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = \frac{\partial w}{\partial \theta_2} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = \frac{\partial p_3}{\partial \theta_2} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)}. \quad (24)$$

In the same way, I can show the following hold:

$$\frac{\partial w}{\partial \rho} = \frac{\{1 - (1+r)a_{11}\}}{l_1} \frac{\partial p_1}{\partial \rho}, \quad (25)$$

$$\frac{(1+r)l_1a_{12}(\theta_2) + l_2(\theta_2)\{1 - (1+r)a_{11}\}}{l_1} \frac{\partial p_1}{\partial \rho} = \theta_2 \frac{\partial p_3}{\partial \rho}, \quad (26)$$

and

$$\begin{aligned} & \left[\frac{(1+r)l_1a_{13}^+(\rho, \theta_4) + l_3^+(\rho, \theta_4)\{1 - (1+r)a_{11}\}}{l_1} \right. \\ & \quad \left. + \frac{\{(1+r)a_{33}^+ - \theta_4\}[(1+r)l_1a_{12}(\theta_2) + l_2(\theta_2)\{1 - (1+r)a_{11}\}]}{\theta_2 l_1} \right] \frac{\partial p_1}{\partial \rho} \\ & = - \left[(1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \rho} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \rho} \right]. \end{aligned} \quad (27)$$

Since the right-hand side of (27) is zero at $(\theta_2^*, \rho^*, \theta_4^*)$ thanks to (17) and $(1+r)a_{33}^+ - \theta_4 > 0$ due to Assumption 4, considering (25) and (26), we know the following holds:

$$0 = \frac{\partial p_1}{\partial \rho} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = \frac{\partial w}{\partial \rho} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = \frac{\partial p_3}{\partial \rho} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)}. \quad (28)$$

Completely in the same way, I can show the following holds:

$$\frac{\partial w}{\partial \theta_4} = \frac{\{1 - (1+r)a_{11}\}}{l_1} \frac{\partial p_1}{\partial \theta_4}, \quad (29)$$

$$(1+r) \frac{\partial p_3}{\partial \theta_4} = \frac{(1+r)a_{12}l_1 + l_2(\theta_2)\{1 - (1+r)a_{11}\}}{l_1\theta_2} \frac{\partial p_1}{\partial \theta_4}, \quad (30)$$

and

$$\begin{aligned}
& \left[\frac{(1+r)l_1 a_{13}^+(\rho, \theta_4) + l_3^+(\rho, \theta_4) \{1 - (1+r)a_{11}\}}{l_1} \right. \\
& \quad \left. + \frac{\{(1+r)a_{33}^+ - \theta_4\} [(1+r)l_1 a_{12}(\theta_2) + l_2(\theta_2) \{1 - (1+r)a_{11}\}]}{\theta_2 l_1} \right] \frac{\partial p_1}{\partial \rho} \\
& = - \left[(1+r)p_1 \frac{\partial a_{13}^+(\rho, \theta_4)}{\partial \theta_4} + w \frac{\partial l_3^+(\rho, \theta_4)}{\partial \theta_4} \right].
\end{aligned} \tag{31}$$

Since the right-hand side of (31) is zero at $(\theta_2^*, \rho^*, \theta_4^*)$ thanks to (17) and $(1+r)a_{33}^+ - \theta_4 > 0$ due to Assumption 4, considering (29) and (30), we know the following holds:

$$0 = \frac{\partial p_1}{\partial \theta_4} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = \frac{\partial w}{\partial \theta_4} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = \frac{\partial p_3}{\partial \theta_4} \Big|_{(\theta_2^*, \rho^*, \theta_4^*)}. \tag{32}$$

From (24), (28) and (32), we have $\Phi(\theta_2, \rho, \theta_4) \Big|_{(\theta_2^*, \rho^*, \theta_4^*)} = 0$. ■

C Appendix

A case in which a_{33} and a_{44} depend upon ρ and (ρ, θ_4) respectively

In the above analysis, I have assumed that a_{33} and a_{44} are given constants for simplicity. This may be justified since how a_{33} and a_{44} react to changes in θ_2 and (ρ, θ_4) is unknown. Yet, one might be interested in a case in which a_{33} and a_{44} depend upon θ_2 and (ρ, θ_4) respectively. Let me briefly consider this possibility.

Suppose that a_{33} depends upon ρ , i.e., it is denoted as $a_{33}(\rho)$. If $da_{33}(\rho)/d\rho > 0$ and $d^2a_{33}(\rho)/d\rho^2 > 0$ hold, the results obtained above still hold. This is the case in which more inputs of ELP are required by the third sector (recycling sector) to enhance the quality of a secondary material and the effect is more than proportionate. This case is more likely than the opposite case, i.e., the case in which $da_{33}(\rho)/d\rho < 0$ and $d^2a_{33}(\rho)/d\rho^2 > 0$ hold¹³.

If one tries to enhance the quality of a secondary material in the third sector, one should increase a yield rate for the enhancement, so that one needs more ELPs for this purpose and must increase the input more than proportionately. This is often seen in recycling fields, as far as I know. If so, the relaxation of the assumption in this line does not change the main results, since the qualitative nature of Ψ^* remains the same as before.

Suppose the opposite case holds. Even so, the main results are still true insofar as $d^2a_{33}(\rho)/d\rho^2 > 0$ holds. Thus, in any case, the results obtained in the previous section are not affected even if I assume a_{33} depends upon ρ .

Next, let me consider the case in which a_{44} depends upon (ρ, θ_4) . First, I would like to examine the effect of ρ on a_{44} . If ρ increases, the input of a secondary material to the fourth sector is possibly saved, thanks to improvement of quality of a secondary material. Then, I can safely assume that $\partial a_{44}(\rho, \theta_4)/\partial \rho < 0$ and $\partial^2 a_{44}(\rho, \theta_4)/\partial^2 \rho > 0$ hold, and so the main results are not affected even if I assume a_{44} depends upon (ρ, θ_4) , since the nature of Ψ^* is unchanged.

¹³Notice that stability conditions are determined by the second-order derivatives of the coefficients, instead of the first-order derivatives. It is most unlikely to see $d^2a_{33}(\rho)/d\rho^2 < 0$ in reality.

Suppose that $\partial a_{44}(\rho, \theta_4)/\partial \rho > 0$ holds, although it is unlikely. Even in this case, the main results are not affected insofar as I assume that $\partial^2 a_{44}(\rho, \theta_4)/\partial \rho^2 > 0$ holds, since the qualitative nature of Ψ^* is not changed. Only in the case in which $\partial^2 a_{44}(\rho, \theta_4)/\partial \rho^2 < 0$ holds and this effect surpasses other terms, making the sign of C_2 positive, the main results are affected¹⁴. However, this case seems most unlikely.

The effect of θ_4 on a_{44} is a little bit subtle. As θ_4 increases, an input of a secondary material may increase or decrease. Both cases are possible, depending upon the character of a secondary material. So, we have $\partial a_{44}(\rho, \theta_4)/\partial \theta_4 \gtrless 0$. However, the main results are the same insofar as $\partial^2 a_{44}(\rho, \theta_4)/\partial \theta_4^2 > 0$ holds. The opposite inequality is very unlikely.

The circumstances become complicated when both the effects of ρ and (ρ, θ_4) on a_{33} and a_{44} are combined, namely when the effects of ρ and θ_4 on $a_{33}^+ \equiv (1+r)a_{33}(\rho)a_{44}(\rho, \theta_4)$ are considered. The effect of θ_4 on a_{33}^+ may be easily accommodated in the main results if $\partial^2 a_{44}(\rho, \theta_4)/\partial \theta_4^2 > 0$ is assumed, since this does not change the nature of Ψ^* . The problem is the effect of ρ on a_{33}^+ .

Let me calculate the second order derivatives as follows:

$$\begin{aligned} \frac{\partial^2 a_{33}^+(\rho, \theta_4)}{\partial \rho^2} &= (1+r) \left[\begin{array}{cccccc} a_{33}''(\rho)a_{44}(\rho, \theta_4) & + & 2a_{33}'(\rho)\frac{\partial a_{44}(\rho, \theta_4)}{\partial \rho} & + & a_{33}(\rho)\frac{\partial^2 a_{44}(\rho, \theta_4)}{\partial \rho^2} \end{array} \right] \gtrless 0 \\ &\quad \begin{array}{cccccc} (+) & (+) & (+) & (-) & (+) & (+) \end{array} \\ \frac{\partial^2 a_{33}^+(\rho, \theta_4)}{\partial \theta_4^2} &= (1+r)a_{33}(\rho) \frac{\partial^2 a_{44}(\rho, \theta_4)}{\partial \theta_4^2} > 0 \\ &\quad \begin{array}{cc} (+) & (+) \end{array} \\ \frac{\partial}{\partial \rho} \left(\frac{\partial a_{33}^+(\rho, \theta_4)}{\partial \theta_4} \right) &= \left[\begin{array}{cccc} a_{33}'(\rho)\frac{\partial a_{44}(\rho, \theta_4)}{\partial \theta_4} & + & a_{33}(\rho)\frac{\partial^2 a_{44}(\rho, \theta_4)}{\partial \rho \partial \theta_4} \end{array} \right] \gtrless 0 \\ &\quad \begin{array}{cccc} (+) & (\pm) & (+) & (+) \end{array} \end{aligned}$$

Then, it is easily seen that a stability/instability nature is affected by $\partial a_{44}(\rho, \theta_4)/\partial \rho$. If this term is large enough to surpass the other terms, a component B_2 of Ψ^* can be positive. Then,

$$\det \Psi^{*(-)} = B_2 C_3 - B_3 C_2 < 0$$

holds. The real part of one of the roots of the characteristic equation is positive, so that the long-run equilibrium point is locally unstable (a saddle point). This result is unaffected by the sign of $\partial a_{44}(\rho, \theta_4)/\partial \theta_4$, although the sign of trace $\Psi^{*(-)}$ is affected.

In summary, I am allowed to say that the stability/instability result is fundamentally the same as the ones obtained in the main text, even if I assume a_{33} and a_{44} depend upon ρ and (ρ, θ_4) respectively. The difference is that a possibility of instability becomes larger under the assumptions in this appendix.

¹⁴In this case, the long-run equilibrium point is stable, since trace $\Psi^* < 0$ det $\Psi^* > 0$

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